

C.E. 418.3 - Reinforced Concrete I

Midterm Examination

November 1, 2000

**UNIVERSITY OF SASKATCHEWAN
DEPARTMENT OF CIVIL ENGINEERING**

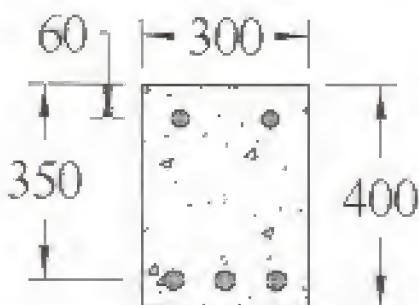
INSTRUCTOR: M. J. Gress

TIME: 2 hours

NOTES: • Students are permitted to use the CPC Concrete Handbook, including CSA Standard A23.3-94.

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QUESTION 1: The reinforced concrete beam shown below was constructed in the 1960's. The reinforcing bar size and concrete and steel strengths are different than used today and are listed below:



- $f_c = 20 \text{ MPa}$
- $f_y = 300 \text{ MPa}$
- $A_s = 3 - \frac{1}{8}^{\text{th}}$ bars
- $A'_{s'} = 2 - \frac{1}{8}^{\text{th}}$ bars

Note: Imperial bar designations are measured in $\frac{1}{8}^{\text{th}}$ inch increments (i.e., $\frac{1}{4}$ bar is $\frac{4}{8}$ dia., $\frac{5}{8}$ bar is $\frac{5}{8}$ dia., etc.)

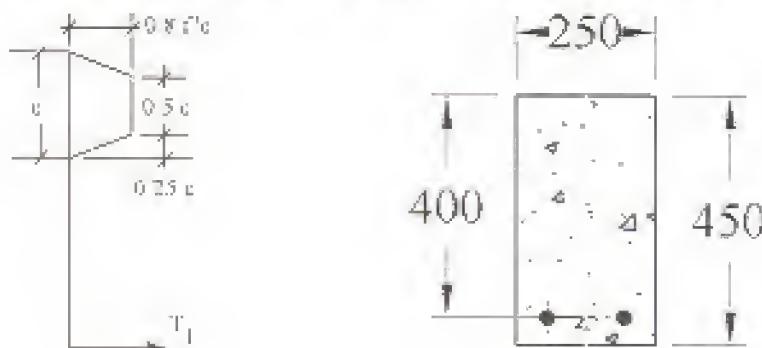
The building in which the above beam is situated in has been subjected to some increased loads. Using current Canadian Design Methods, determine M_r . Perform all necessary calculations as to reinforcement limits and check all assumptions made.

HINT: Assume the top steel does not yield.

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QUESTION 2: A new reinforced concrete code, similar to USD, is being developed in Europe. Unlike the Canadian Code, the European code has no material factors (i.e., ϕ_s and ϕ_c). Also, the "Whitney" stress block is not used, but rather, a triangular shaped stress block is used as shown below.

- $f_c = 35 \text{ MPa}$
- $f_y = 400 \text{ MPa}$
- $A_s = 2000 \text{ mm}^2$



For the beam and stress block shown above, calculate the Moment resistance of the beam. (Remember LSD factors are not applicable, use basic principles.)

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QUESTION 3: Toronto, Ontario, has a bid for the 2008 Olympics. Currently, preliminary design work is underway for some of the facilities. You have been hired to put in a proposal for the design of a concrete 10m diving platform. The Olympic Committee regulates the size and thickness of the deck (see below).

- $f_c = 35 \text{ MPa}$
- $f_t = 400 \text{ MPa}$
- $w_{dead} = 5 \text{ kN/m}$
- $w_{live} = 1.4 \text{ kN/m}$
- Clear cover = 20 mm
- Use 15M bars
- exposure is interior and non-aggressive



For the material properties and loading shown above, calculate and select the reinforcing steel for the cantilevered platform. Design a singly reinforced member (i.e. tension steel only). Once selected, draw a cross section of the beam and draw in the reinforcement you have selected.

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QUESTION 4: Another group has sent in their own proposal following the same loads, material strengths, clear cover and rebar size. However, they have designed it with a doubly reinforced section to take into account the large deflections which are inherent with cantilevers. They have provided 5 – 15M's for both the tension steel and the compression steel.

Determine what the deflection due to long term dead load will be for the structure after 10 years and comment on the extent of the deflection. (Use $I_u = 140.0 \times 10^6 \text{ mm}^4$ do not calculate this value. Also, M_s can be found assuming full live and dead loads.)

QUESTION 1: The reinforced concrete beam shown below was constructed in the 1960's. The reinforcing bar size and concrete and steel strengths are different than used today and are listed below:

- $f_c = 20 \text{ MPa}$
- $f_y = 300 \text{ MPa}$
- $A_s = 3 - 7 \text{ bars}$
- $A'_s = 2 - 8 \text{ bars}$

Note: Imperial bar designations are measured in $\frac{1}{8}$ inch increments (i.e., "4 bar is $\frac{4}{8}$ " dia., "5 bar is $\frac{5}{8}$ " dia., etc.)

The building in which the above beam is situated in has been subjected to some increased loads. Using current Canadian Design Methods, determine M_c . Perform all necessary calculations as to reinforcement limits and check all assumptions made.

$$\text{Given } b = 300 \text{-mm} \quad h = 400 \text{-mm} \quad d' = 60 \text{-mm} \quad d = 350 \text{-mm} \quad f_c = 20 \text{-MPa} \quad f_y = 300 \text{-MPa}$$

$$3 - \text{Bottom Bars} (\#7 \text{ bar} = \frac{7}{8} \text{ inch dia}) \quad A_s = 3 \cdot \frac{\pi \cdot \left(\frac{7}{8} \cdot \frac{\text{in}}{2}\right)^2}{4} \quad A_s = 1163.844 \text{ mm}^2$$

$$2 - \text{Top Bars} (\#8 \text{ bar} = \frac{8}{\pi} \text{ inch dia}) \quad A_s = 2 \cdot \frac{\pi \cdot \left(\frac{8}{\pi} \cdot \text{in}\right)^2}{4} \quad A_s = 1013.415 \text{ mm}^2$$

$$\alpha_1 = 0.85 - 0.0015 \cdot \frac{1}{f_c} \quad \alpha_1 = 0.82 \quad \beta_1 = 0.97 - 0.0025 \cdot \frac{1}{f_c} \quad \beta_1 = 0.92$$

LSD Material Factors $\phi_c = 0.6$ $\phi_i = 0.85$

Compression Forces

$$C_1 = \alpha_1 \cdot \phi_{\epsilon}(\{e\}; a, b) \quad \text{where} \quad a = \beta_1 \cdot e \quad T_1 = \phi_{\epsilon}(\{e\}; A_1)$$

$$C_1 = \alpha_1 \cdot \phi_{\psi} \cdot f_{\psi} \cdot \beta_1 \cdot e \cdot b \quad T_2 = \phi_{\psi} \cdot f_{\psi} \cdot \Lambda_{\psi} \cdot \varphi$$

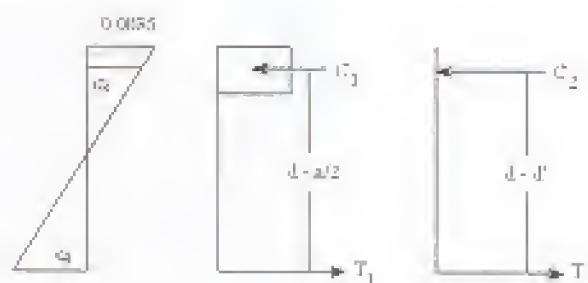
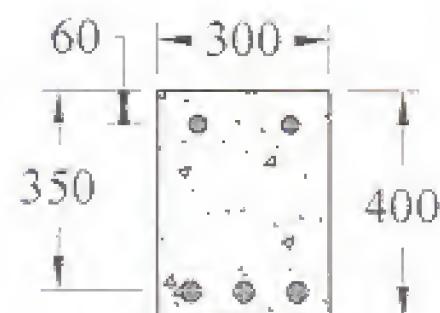
$$C_2 = A_{\pi'} \left(\phi_{\pi'} f_s - \alpha_{\pi'} \phi_{\pi'} f_c \right) \quad T = T_1 + T_2$$

Tension Forces

$$T_1 = \phi_s \cdot E_s A_{s,1}$$

$$T_2 = \phi_s \cdot t_Y \cdot A_{s,2}$$

$$T = T_1 + T_2$$



Assume $\epsilon'_s < \epsilon_y$ and then verify.

From Strain Diagram

$$\frac{\epsilon'_s}{c-d} = \frac{0.0035}{c}$$

Therefore $\epsilon'_s = 0.0035 \cdot \frac{c-d}{c}$ and $f_s = E \cdot \epsilon'_s$ $f_s = E \left(0.0035 \cdot \frac{c-d}{c} \right)$ $E = 200000 \text{ MPa}$

Find neutral axis location - c

$$\sum F_H = 0 \quad \text{Therefore} \quad T = C_1 + C_2 \quad \phi_s f_y A_s = (\alpha_1 \phi_c f_c \beta_1 c b) + A'_s (\phi_s f_s - \alpha_1 \phi_c f_c)$$

Given $\phi_s f_y A_s = (\alpha_1 \phi_c f_c \beta_1 c b) + A'_s \left[\phi_s \left[E \left(0.0035 \cdot \frac{c-d}{c} \right) \right] - \alpha_1 \phi_c f_c \right]$ $c = \text{Find}(c) \quad c = 73.117 \text{ mm}$

Check that top steel has not yielded as per our assumption. $f_s = E \left(0.0035 \cdot \frac{c-d}{c} \right) \quad f_s = 125.582 \text{ MPa} < f_y = 300 \text{ MPa} \quad \text{Therefore OK!}$

Verify result

$$C_1 = \alpha_1 \phi_c f_c \beta_1 c b \quad C_2 = A'_s (\phi_s f_s - \alpha_1 \phi_c f_c) \quad T = \phi_s f_y A_s$$

$$C_1 = 198.575 \text{ kN} \quad C_2 = 98.205 \text{ kN} \quad T = 296.78 \text{ kN} \quad C_1 + C_2 = 296.78 \text{ kN}$$

Check that $\epsilon_s > \epsilon_y$ by checking Clause 10.5.2

$$\frac{c_{\max}}{d} = \frac{700}{700 + \frac{f_y}{\text{MPa}}} \quad c_{\max} = \frac{700}{700 + \frac{f_y}{\text{MPa}}} \cdot d \quad c_{\max} = 245 \text{ mm} \quad c = 73.117 \text{ mm} \quad \text{Therefore OK!}$$

Now find the factored Moment Resistance, LSD $a = \beta_1 c$

$$M_r = C_1 \left(d - \frac{a}{2} \right) + C_2 (d - d') \quad M_r = 91.302 \text{ kN}\cdot\text{m}$$

Find min steel allowed Clause 10.5.1

$$A_{smin} = \frac{0.2 \cdot \sqrt{\frac{f_c}{f_y}} \cdot b \cdot h}{\frac{f_y}{\text{MPa}}} \quad A_{smin} = 357.771 \text{ mm}^2$$

$$A_s = 1163.844 \text{ mm}^2 \quad \text{We are within the limits!!}$$

25 **QUESTION 2:** A new reinforced concrete code, similar to USD, is being developed in Europe. Unlike the Canadian Code, the European code has no material factors (i.e., s and c). Also, the "Whitney" stress block is not used, but rather, a triangular shaped stress block is used as shown below.

For the beam and stress block, calculate the Moment resistance of the beam. (Remember LSD factors are not applicable, use basic principles.)

Given $f_c = 35 \text{ MPa}$ $f_y = 400 \text{ MPa}$ $b = 250 \text{ mm}$ $d = 400 \text{ mm}$ $h = 450 \text{ mm}$

$$A_s = 2000 \text{ mm}^2$$

Tensile Force $T = A_s f_y$ $T = 800 \text{ kN}$

Top Triangular Compressive Force $C_1 = \frac{1}{2} \cdot (0.8 \cdot f_c \cdot b) \cdot \frac{c}{4}$

Middle Compressive Force $C_2 = (0.8 \cdot f_c \cdot b) \cdot \frac{c}{2}$

Bottom Triangular Compressive Force $C_3 = \frac{1}{2} \cdot (0.8 \cdot f_c \cdot b) \cdot \frac{c}{4}$

Given $T = \frac{1}{2} \cdot (0.8 \cdot f_c \cdot b) \cdot \frac{c}{4} + (0.8 \cdot f_c \cdot b) \cdot \frac{c}{2} + \frac{1}{2} \cdot (0.8 \cdot f_c \cdot b) \cdot \frac{c}{4}$ $c = \text{Find}(c)$ $c = 152.381 \text{ mm}$

Now Calculate the Compressive Force Components and Calculate the Moment Capacity

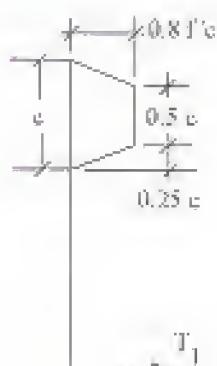
$$C_1 = \frac{1}{2} \cdot (0.8 \cdot f_c \cdot b) \cdot \frac{c}{4} \quad C_1 = 133.333 \text{ kN}$$

$$C_2 = (0.8 \cdot f_c \cdot b) \cdot \frac{c}{2} \quad C_2 = 533.333 \text{ kN}$$

$$C_3 = \frac{1}{2} \cdot (0.8 \cdot f_c \cdot b) \cdot \frac{c}{4} \quad C_3 = 133.333 \text{ kN}$$

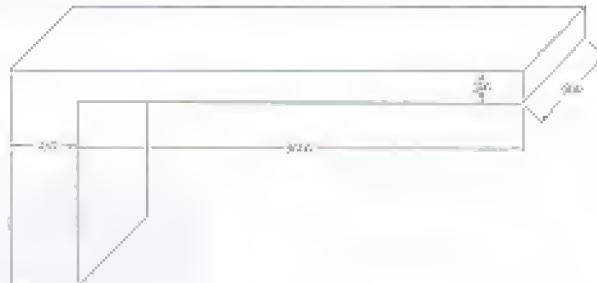
$$C_1 + C_2 + C_3 = 800 \text{ kN} \quad T = 800 \text{ kN}$$

$$M = C_1 \left(d - \frac{2}{3} \cdot \frac{c}{4} \right) + C_2 \left(d - \frac{c}{2} \right) + C_3 \left(d - c + \frac{2}{3} \cdot \frac{c}{4} \right) \quad M = 259.048 \text{ kN}\cdot\text{m}$$



QUESTION 3: Toronto, Ontario, has a bid for the 2008 Olympics. Currently, preliminary design work is underway for some of the facilities. You have been hired to put in a proposal for the design of a concrete 10m diving platform. The Olympic Committee regulates the size and thickness of the deck (see below).

- $f_c = 35 \text{ MPa}$
- $f_y = 400 \text{ MPa}$
- $w_{\text{dead}} = 3 \text{ kN/m}$
- $w_{\text{live}} = 1.4 \text{ kN/m}$
- Clear cover = 20 mm
- Use 15M bars
- exposure is interior and non-aggressive



For the material properties and loading shown above, calculate and select the reinforcing steel for the cantilevered platform. Design a singly reinforced member (i.e. tension steel only). Once selected, draw a cross section of the beam and draw in the reinforcement you have selected.

$$\text{Given } b = 600 \text{ mm} \quad h = 200 \text{ mm} \quad f_y = 400 \text{ MPa} \quad f_c = 35 \text{ MPa} \quad \gamma_c = 2400 \frac{\text{kg}}{\text{m}^3} \quad \lambda = 1$$

$$\phi_c = 0.6 \quad \phi_s = 0.85 \quad L = 3 \text{ m}$$

$$\alpha_1 = 0.85 - 0.0015 \cdot f_c \cdot \frac{1}{\text{MPa}} \quad \alpha_1 = 0.7975 \quad \beta_1 = 0.97 - 0.0025 \cdot f_c \cdot \frac{1}{\text{MPa}} \quad \beta_1 = 0.8825$$

$$\text{Dead Load} \quad w_D = 3 \frac{\text{kN}}{\text{m}} \quad \text{Live Load} \quad w_L = 1.4 \frac{\text{kN}}{\text{m}}$$

$$\text{Factored Load} \quad w_f = 1.25 \cdot w_D + 1.5 \cdot w_L \quad w_f = 5.85 \frac{\text{kN}}{\text{m}} \quad M_f = \frac{w_f \cdot L^2}{2} \quad M_f = 26,325 \text{ kN}\cdot\text{m}$$

$$\text{Knowing } C = T \text{ and } T = \phi_s f_y A_s \quad \text{and} \quad C = \alpha_1 \phi_c f_c a b \quad \text{And that} \quad M_t = M_f = T \left(d - \frac{a}{2} \right)$$

$$\text{We must find } A_s \quad d = h - 20 \text{ mm} - \frac{16 \text{ mm}}{2} \quad d = 172 \text{ mm}$$

$$\text{Rearranging } C=T \quad A_s = \frac{\alpha_1 \phi_c f_c a b}{\phi_s f_y} \quad \text{and substituting into} \quad M_f = \phi_s f_y A_s \left(d - \frac{a}{2} \right)$$

$$\text{Given} \quad M_f = \phi_s f_y \frac{\alpha_1 \phi_c f_c a b}{\phi_s f_y} \left(d - \frac{a}{2} \right) \quad a = \text{Find}(a) \quad a = 15.973 \text{ mm}$$

$$\text{Therefore} \quad A_s = \frac{\alpha_1 \phi_c f_c a b}{\phi_s f_y} \quad A_s = 472.074 \text{ mm}^2$$

Using 15M bars, 3 bars will work $A_s = 600 \text{ mm}^2$

Check that $c_s > c_y$ by checking Clause 10.5.2

$$\frac{c_{\max}}{d} = \frac{700}{700 + \frac{f_y}{MPa}}$$
$$c_{\max} = \frac{700}{700 + \frac{f_y}{MPa}} \cdot d$$
$$c_{\max} = 109.455 \text{ mm}$$
$$c = \frac{a}{\beta_1}$$
$$c = 18.1 \text{ mm}$$

Therefore OK!

Find min steel allowed Clause 10.5.1

$$A_{s\min} = \frac{0.2 \cdot \sqrt{\frac{f_e}{MPa}} \cdot b \cdot h}{\frac{f_y}{MPa}}$$
$$A_{s\min} = 354.965 \text{ mm}^2$$
$$A_s = 600 \text{ mm}^2$$

We are within the limits!!!

QUESTION 4: Another group has sent in their own proposal following the same loads, material strengths, clear cover and rebar size. However, they have designed it with a doubly reinforced section to take into account the large deflections which are inherent with cantilevers. They have provided 5 - 15M's for both the tension steel and the compression steel.

Determine what the deflection due to long term dead load will be for the structure after 10 years and comment on the extent of the deflection. (Use $I_{cr} = 140.0 \times 10^6 \text{ mm}^4$ do not calculate this value. Also, M_a can be found assuming full live and dead loads.)

$$\text{Given } b = 600 \cdot \text{mm} \quad h = 200 \cdot \text{mm} \quad f_y = 350 \cdot \text{MPa} \quad f_c = 35 \cdot \text{MPa} \quad \gamma_e = 2400 \cdot \frac{\text{kg}}{\text{m}^3} \quad \lambda = 1$$

$$\phi_c = 0.6 \quad \phi_e = 0.85 \quad L = 3 \cdot \text{m}$$

$$d = h - 20 \cdot \text{mm} - \frac{16 \cdot \text{mm}}{2} \quad d' = 20 \cdot \text{mm} + \frac{16 \cdot \text{mm}}{2} \quad d = 172 \cdot \text{mm} \quad d' = 28 \cdot \text{mm}$$

$$\text{Dead Load} \quad w_D = 3 \cdot \frac{\text{kN}}{\text{m}}$$

$$\text{Live Load} \quad w_L = 1.4 \cdot \frac{\text{kN}}{\text{m}}$$

$$A_s = 1000 \cdot \text{mm}^2 \quad A'_s = 1000 \cdot \text{mm}^2$$

$$\text{Specified Moment} \quad M_a = \frac{(w_D + w_L) \cdot L^2}{2} \quad M_a = 19.8 \text{ kN} \cdot \text{m} \quad E_s = 200000 \cdot \text{MPa} \quad E_c = 4500 \cdot \sqrt{f_c \cdot \text{MPa}} \quad E_c = 26622.359 \text{ MPa}$$

$$\text{Find the Transformed Section:} \quad A_s = 1000 \text{ mm}^2 \quad A'_s = 1000 \text{ mm}^2 \quad \alpha = \frac{E_s}{E_c} \quad n = 7.512$$

$$\begin{aligned} \text{Transformed Areas} \quad A'_t &= A'_s \cdot (n - 1) & A'_t &= 6512.482 \text{ mm}^2 \\ A_t &= A_s \cdot n & A_t &= 7512.482 \text{ mm}^2 \end{aligned}$$

Summation of 1st moments of areas (i.e., $y = 0$, centred)

$$\text{Given } b \cdot c \cdot \frac{c}{2} + A'_t(c - d') = A_t(d - c) \quad c = \text{Find}(c) \quad c = 50.526 \text{ mm}$$

Transformed (cracked) Moment of Inertia

$$I_{cr} = \frac{b \cdot c^3}{3} + A_t(d - c)^2 + A'_t(c - d')^2 \quad I_{cr} = 139.96 \times 10^6 \text{ mm}^4$$

Gross Moment of Inertia

$$I_g = \frac{1}{12} \cdot b \cdot h^3 \quad I_g = 400000000 \text{ mm}^4$$

Cracking Moment and Effective Moment of Inertia $y_t = \frac{h}{2}$ $y_t = 100 \text{ mm}$ $f_r = 0.6 \cdot \lambda \cdot \sqrt{f_c' \text{ MPa}}$

$$M_{cr} = f_r \frac{l_g}{y_t} \quad M_{cr} = 14.199 \text{ kN}\cdot\text{m}$$

$$I_e = I_{sr} + (I_s - I_{sr}) \cdot \left(\frac{M_{cr}}{M_u} \right)^3 \quad I_e = 235.85 \cdot 10^6 \cdot \text{mm}^4 \quad \frac{I_{sr}}{I_e} = 59.341\%$$

Deflection after attachment of non-structural elements (Clauses 9.8.2.5, 9.8.2.6 and Table 9.2)

$$\rho' = \frac{A'_s}{b \cdot d} \quad \rho' = 0.969\% \quad S = 2$$

Long term due to (w_0) $\Delta_{\text{Long_Term}} = \frac{w_0 \cdot L^4}{8 \cdot E_e \cdot I_e} \cdot \frac{S}{1 + 50 \cdot \rho'} \quad \Delta_{\text{Long_Term}} = 6.518 \text{ mm}$

$$\frac{L}{\Delta_{\text{Long_Term}}} = 460.293$$

Comment on Deflection: The deflection is only 1/4 of an inch. Which appears to be acceptable.